Chapter 4 - The Case of Low Resistivity Plasmas

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4.1 Introduction

For the plasma described in the final chapter of this thesis, the theoretical values of the damping parameter, $\Omega_e \tau$, range from several hundred to a value of ten thousand. This represents very low damping and unusual effects occur at the plasma vacuum boundary. This chapter presents an investigation into the regions of validity for the 'helicon' approximation and the hydrodynamic approximation where the plasma is assumed to be cold.

As mentioned in the previous chapter, discontinuities appear in certain field components at the interface of a plasma and a rigid non-conductor when the resistivity, $\eta \to 0$. If the plasma is bounded by perfectly conducting walls, surface currents cannot exist and the dispersion relation can be simply solved.

4.2 Experimental Considerations

Although the problem of solving the dispersion relation with non-conducting walls has been discussed by Stix (1962), Woods (1962), and Bernstein and Trehan (1960), the case of $\eta \to 0$ was not treated very fully. The main papers referred to in this chapter will be Klozenberg, McNamara, and Thonemann (KMT) (1965), Legendy (1964), Woods (1964), and Lehane and Thonemann (1965).

As the magnitude of the surface currents for non-conducting boundaries is unknown, the boundary conditions for zero resistivity
cannot be obtained, but by allowing $\Omega_e^\tau$ to tend to infinity the radial variation of the wave fields in the neighbourhood of the interface can be obtained. For reasons of theoretical simplicity KMT consider the $m = 0$ azimuthal mode for a uniform cylindrical plasma for waves in the frequency range $\Omega_1^i << \omega << \Omega_e^\tau$.

As $\Omega_e^\tau$ became large they found that the resistive effects were finite only in a region of thickness

$$\delta \sim \frac{1}{k\Omega_e^\tau},$$

which degenerates to a discontinuous function $d(a-r)$, as $\Omega_e^\tau \to \infty$

where $d(a-r) = 1$ if $a = r$

$= 0$ otherwise.

In this case the $\theta$ and $z$ components of the magnetic field had finite discontinuities while the radial component remained continuous.

Francey and Gates (1968) have also considered this situation but conclude that surface currents do not exist. Davies (1969) has pointed out an internal inconsistency in their paper arising from their neglect of the wave field $E_r$ which becomes infinite at the interface. The introduction of a dipole layer at the surface, as suggested by Woods (1962, 1964) can explain this phenomena. It is the purpose of this chapter to show that the limit $\delta = 0$ is not theoretically attainable, nor is it experimentally attainable. The results of Lehane and Thonemann (L & T) are used to show that although the resistive dissipation may be finite only in a small surface layer for the helicon approximation,
this approximation is invalid for low values of the resistivity and the effect of electron inertia must be included. In this low resistivity region, the thermal effects of the plasma, such as finite Larmor radius, set a lower limit for $\delta$.

For a plasma source L & T used a R.F. oscillator at 15-17 MHz to excite Xenon gas to about 1% ionization. The electron neutral collision frequency was changed by changing the neutral gas pressure. Although their experiments verified the predictions of the KMT theory for frequencies $\Omega_i \ll \omega \ll \Omega_e$, the 'anomalous' damping in this frequency region predicted by KMT was not observed and a simple analysis shows that this 'anomalous' damping when $n \to 0$ does not exist.

To lower the collision frequency it is necessary to decrease the neutral gas pressure and consequently the charged particle density. The characteristic frequency $\omega_o = \frac{B_0}{4\pi n e a^2}$ increases and to keep $ak$ (the dimensionless wave number) constant, the wave frequency must be increased. If the axial magnetic field is kept constant, the wave frequency will approach the electron-cyclotron frequency $\Omega_e$ as $n$ and consequently $n_0$, the charged particle density is decreased. The helicon approximation therefore becomes invalid as $n$ decreases, requiring the inclusion of the electron inertial term in the dispersion relation. Eventually as $n \to 0$ the frequency of the wave becomes higher than $\Omega_e$ and the wave becomes evanescent.

To examine the effect on the wave fields of the $m = 1$ mode of decreasing $n$, the computer program described in chapter 3 was used. Since the boundary conditions and method of solution differ slightly from that of KMT, a number of dispersion curves for differing $\Omega_e$,
Fig. 4.1(a)  Dispersion curves for $m = 1$ standing helicon waves with $\Omega_e/\omega_o = 0$. 
RIGID NONCONDUCTING BOUNDARIES

PLANE WAVE

CONDUCTING BOUNDARIES
Fig. 4.1(b) Damping of $m = 1$ standing helicon waves for different values of $\Omega_e \tau$ with rigid non-conducting cylindrical boundaries.
neglecting electron inertia were calculated. These showed excellent agreement with KMT. The experimental results of L and T were then verified theoretically for values of $\Omega_e \tau$ up to 30 (L and T figure 7). Hence it was concluded that the theory described in section 3.5 accurately predicts the dispersion of small amplitude helicon waves in a plasma surrounded by insulating walls. Representative dispersion curves for $M = 1$ standing helicons are shown in figure 4.1 (a) and 4.1 (b). The wave fields at higher $\Omega_e \tau$ were investigated assuming resistivity proportional to neutral gas pressure and a percentage ionization of 1%.

The results for eight different radial solutions is summarized in figure 4.2. The real part of the dimensionless wave number, $a_k$, was kept constant at 0.68 and the product of $\Omega_e \tau$ and $\Omega_e / \omega_0$ (the electron inertial term) kept constant at 7,200, (since $\Omega_e \tau \times \Omega_e / \omega_0 = \left(\frac{4\pi e^2 a^2 B}{m^2}\right)n\tau$ i.e. $= n\tau$, and the collision time $\tau$ has been assumed to increase inversely proportionally to $n$, therefore $(\Omega_e \tau) \times \Omega_e / \omega_0$ is constant). These roots of the dispersion relations were found by estimating the imaginary part of the wave number and the real frequency and minimizing these values using the computer program. Roots were then followed by slightly changing the values of $\Omega_e / \omega_0$ and $\Omega_e \tau$. The separate radial solutions could be found by examining the computed graphs of the wave fields for extreme values of $\Omega_e / \omega_0$ and $\Omega_e \tau$ where one of these parameters was far larger than the other. In the intermediate region, when the values were comparable, the wave fields could not be defined by integer values of the radial mode number. For example A and B in figure 4.2 can easily be seen to be describing the $n = 1$ radial mode for large or small values of $\Omega_e \tau$, gradually change to different radial modes. Computed wave fields for
Fig. 4.2 Dispersion curves of frequency as a function of $\Omega_e \tau$ and $\Omega_e / \omega_o$ with $ak = 0.68$ for $m = 1$ travelling helicon waves with rigid non-conducting cylindrical boundaries.
Fig. 4.3 Magnetic field components as a function of radius for different values of $\Omega_e \tau$ along curve A of Fig. 4.2.
Fig. 4.4  Magnetic field components as a function of radius for different values of $\Omega_e$ along curve B of Fig. 4.2.
these curves are shown in figures 4.3 and 4.4. Curve A, which describes the dispersion of the waves experimentally examined by L & T changed from \( n = 1 \) to \( n = 4 \) at \( \Omega_e \tau \sim 120 \). Curve B describes the dispersion of a wave mode which starts with the most simple radial structure \( n = 1 \) for high values of \( \Omega_e \tau \), gradually changes to \( n = 2 \) at \( \Omega_e \tau \sim 200 \) and \( n = 3 \) at \( \Omega_e \tau \sim 100 \).

It was found that the simplest radial mode \( n = 1 \) does not propagate for quite a large range of values of \( \Omega_e \tau \) from 120 to 300. Although the effect of the electron inertial term can be easily seen, all frequencies on the graph (figure 4.2) are at least an order of magnitude below the electron-cyclotron frequency.

At this stage a comparison with waves of the same wavelength in a cylinder with conducting walls shows some important differences. Computed results for a resistiveless plasma \( (v<<\omega) \) for a similar range of \( \Omega_e/\omega_o \) are given in figure 4.5. Included in both graphs is the region where solutions of the dispersion relation are not allowed (see chapter 3, section 4 of this thesis). With conducting walls, the tangential components of the electric field must be zero for all frequencies, and a radial wave number can be simply defined. Although the first radial mode can propagate for all values of \( \Omega_e/\omega_o \), there are certain values of this parameter where more than one mode can propagate. These frequencies are defined by the points of intersection of the dispersion curves. The effect of including resistivity is shown in figure 4.6. Higher order radial modes suffer greater damping and the dispersion curves effectively pass over each other in this three-dimensional representation.

This does not occur with non-conducting walls however, and the ordering of the radial modes is achieved by the gradual change along the
Fig. 4.5 Dispersion curves of frequency as a function of $\Omega_e/\omega_o$ with $ak = 0.68$ for a resistiveless plasma for $m = 1$ travelling helicon waves with conducting cylindrical boundaries.
Fig. 4.6 Dispersion curves of frequency as a function of $\Omega_e \tau$ with $ak = 0.68$ for $m = 1$ travelling helicon waves with conducting cylindrical boundaries showing effect of damping.
the dispersion curve of the radial field complexity. As the boundary conditions in this case require only a matching to outside fields, the field components are not restricted to specific values at the interface. The concept of an integer radial mode number therefore becomes meaningless except in cases where \( \Omega_e \tau \) and \( \Omega_e / \omega_o \) are very large or very small. It would appear that in the region where \( \Omega_e \tau \) and \( \Omega_e / \omega_o \) are comparable and equal to approximately 100, significant differences would exist for wave propagation with the two different boundary conditions.

Physical reasons for this behaviour have not been found yet, but some information can be obtained by looking more closely at the effect of the electron inertial term. It has been shown by Woods (1964) that the inclusion of electron inertia adds a term \( \frac{m_e \beta j}{ne^2 \beta t} \) to \( \eta j \) in Ohms law and for oscillatory perturbations the coefficient of \( j \) is changed from \( \eta \) to \( \eta - i \left( \frac{\omega m_e}{ne^2} \right) \). KMT use a length \( \delta \) to characterize the distance over which resistive effects are finite for small \( \eta \) whereas Woods uses dimensionless parameters involving \( (k^2 \delta')^{1/2} \) where \( \delta' \) has the dimensions of length squared. If inertial effects are neglected \( \delta' \to 0 \) as \( \eta \to 0 \) and is consequently similar to the 'skin depth' \( \delta \) defined by KMT. By including electron inertial effects, Woods derives

\[
\delta' = \left( \frac{4 \pi \omega}{\eta} \right)^{-1} - i \left( \frac{c}{\omega_p} \right) \tag{4.1}
\]

The imaginary part of \( \delta' \) can be interpreted as an evanescent component of a radial wavelength. In the limit \( \eta \to 0 \), \( \delta' \to -i \left( \frac{c}{\omega_p} \right) \) and the limit \( \delta' = 0 \) is not theoretically attainable, precluding any surface currents from flowing.
Rearranging equation 4.1 and using the plane wave helicon approximation:

\[ \delta' = (k^2 \Omega_e \tau)^{-1} - i \left( \frac{\omega a^2}{\Omega_e} \right). \tag{4.2} \]

We would expect resistive effects to cease being dominant when:

\[ (k^2 \Omega_e \tau)^{-1} = \frac{\omega a^2}{\Omega_e}. \tag{4.3} \]

For the plasma described by L & T this occurs when \( \Omega_e \tau = 120 \).

For \( \Omega_e \tau > 120 \) (i.e. \( \Omega_e / \omega < 60 \)) electron inertial effects will dominate the wave damping, as can be seen from figure 4.2. In this specific experiment, the theory of KMT is therefore invalid for \( \Omega_e \tau > 120 \).

The mechanism of surface losses has been discussed by Legendy (1964). Most authors postulate the existence of a dipole layer just inside the surface consisting of surface charges on the boundary and a space charge of opposite sign exponentially falling off toward the interior of the plasma. Between the two charge layers there is a strong electric field in the \( \tau \) direction which interacts with the axial magnetic field to give a strong current sheet tangential to the interface. The non-zero imaginary part of \( k \) which KMT have shown to be non-zero as \( n \to 0 \) is due to this surface dipole charge layer (Davies (1969)).

### 4.3 Collisionless Damping Mechanisms

The inclusion of thermal effects into the description of a plasma makes the description of the dispersion and attenuation of waves extremely
complicated. This problem has been investigated by Bernstein (1958) and many others, and involves the use of the Boltzmann equation to determine the wave fields. In gaseous plasmas a Maxwellian distribution of electron velocities is generally assumed whereas a Fermi distribution of charge carriers is assumed in solid state semiconductor plasmas. Since the electrons have a spread of energies, the dispersion relation cannot be regarded as having unique solutions. Approximation methods are generally employed by expanding wave parameters as power series. In this discussion only first order terms will be retained and the imaginary parts of certain parameters (e.g. N and k) are assumed small compared to the real parts.

The simple inclusion of a finite Larmor radius for the electrons shows the limitations of the hydrodynamic model for a plasma. When the electron Larmor radius becomes greater than the skin depth $\delta$, the strong electric field between the two charge layers has no first order effect on the conduction. Thus it cannot bring about the strong currents responsible for the surface loss, which consequently disappears (Legéndy). As $n \to 0$, the collisionless damping mechanism becomes important with the wave energy being transferred directly to the charged particles of the plasma. The collisionless damping of plasma waves by electrons moving with a velocity close to the phase velocity of the wave is called Landau damping. This collisionless mechanism was first investigated by Landau (1946) who considered electrons becoming trapped in the potential wells of the wave electric fields. Since gaseous plasmas have electron velocity distributions with a high energy tail there will be electrons with velocities greater than the phase velocity of the wave. This situation is analogous to that of Cherenkov radiation, where absorption of the
wave energy due to these electrons is called Cherenkov absorption. This mechanism has been theoretically treated by Shafranov (1958). Obviously, electrons will also absorb energy from the right-hand polarized wave at the electron-cyclotron frequency. In general, electrons will contribute to absorption when

\[ \omega - n | \Omega_e | - k_z v_z = 0 \]  \hspace{1cm} (4.4)

where \( n = 0, \pm 1, \pm 2, \ldots \).

and \( v_z \) and \( k_z \) are the electron velocity and plasma wave number in the direction of \( B_0 \) (assuming \( v_z << c \)).

This relation can be seen to be the Doppler formula, which relates the frequency of a radiator \( n \Omega_e \) (in the particle rest system) to the frequency \( \omega \) which is observed in the laboratory coordinate system. If \( n < 0 \), the effect is called the "anomalous" Doppler effect. When \( n > 0 \) we find the frequencies \( \omega \) at which the particle moving with velocity \( v_z \) along the lines of force is in resonance with the wave as a consequence of the Doppler effect. In this case the absorption is called Doppler shifted cyclotron absorption which for a cold plasma reverts simply to

\[ \omega = n \Omega_e . \]

Cherenkov damping occurs when \( n = 0 \), since the absorption condition \( \omega = k_z v_z \) coincides with the Cherenkov radiation condition for a charge moving with a fixed velocity along the \( z \)-axis.
As the Cherenkov and cyclotron damping mechanisms are both collisionless, it is important to this discussion to show in which frequency regions one or the other might be expected to dominate. We will assume a Maxwellian distribution of electrons and a wave travelling through an infinite uniform plasma at an angle $\theta$ to the magnetic field $B_0$.

The number of charged particles that contribute to the absorption of the wave energy is proportional to $\exp(-m_e v_z^2/2T)$. Using equation (4.4):

$$\exp(-m_e v_z^2/2T) = \exp(-m_e (\omega - \omega_e)^2/2k_z^2T) \quad (4.5)$$

where $v_z$ is the velocity of the absorbing particles parallel to $B_0$.

$m_e$ is the mass of the electrons,

$T$ is the temperature of the particles measured in units of energy.

For the right-hand polarized wave travelling at an angle $\theta$ relative to $B_0$ and assuming $\omega_p^2 >> \omega_e^2$ and $\theta$ not close to $\pi/2$:

$$N^2 = \frac{\omega_p^2}{\omega (\Omega_e - \cos \theta - \omega)} \quad (4.6)$$

for the frequency region $\Omega_e << \omega < \Omega_e$ for the hydromagnetic approximation.

The inclusion of thermal effects leads to an imaginary part of $N^2$ and a small correction to the real part of $N^2$. The correction to the real part of $N^2$ leads to a slight change in the phase velocity of the wave ($v_\phi = \frac{E}{N}$) which will be neglected since we are only interested in the damping. Thus for the Cherenkov absorption mechanism to be important $v_\phi >> v_T \left[\frac{2T}{m_e}\right]^{1/2}$ (Shafranov 1967).
If $\omega$ is close to $\Omega_e$, both the Cherenkov and cyclotron absorption mechanisms will occur, with the cyclotron absorption dominating as $\omega \to \Omega_e$.

The cyclotron absorption is appreciably smaller than the Cherenkov absorption when the exponential factor $\exp\left(\frac{(\omega - \Omega_e)^2}{k^2 v_F^2}\right)$ (due to cyclotron mechanism) is comparable to $\exp\left(-\frac{\omega^2}{k^2 z v_T^2}\right)$ (due to Cherenkov mechanism).

i.e. when $\omega < \frac{1}{2} \Omega_e$

To find the damping of the wave due to Cherenkov mechanism we assume the refractive index is complex, $N = N_r + iN_i$ where $N_r$ is the refractive index in a cold plasma and $N_i \ll N_r$. From this assumption (Shafranov (1967))

$$N^2 = N_r^2 \left[1 + \sqrt{\pi} \frac{\sin^2 \theta}{| \cos \theta |} \frac{\omega}{\Omega_e} \phi(z)\right] \quad (4.7)$$

where $z = (\beta N_r \cos \theta)^{-1}$

$\beta = v_F/c$

and $\phi(z) = z^3 \exp(-z^2) \quad (4.8)$

It should be remembered that we have assumed $N_i \ll N_r$ which implies $z \ll 1$ yielding the form for $\phi(z)$ in equation 4.8. The general form for $\phi(z)$ for all values of $z$ is given by Stepanov (1961) and is given in a graphical form for $0 \leq z \leq 3$ by Lominadze and Stepanov (1969) for a degenerate Fermi gas in semiconductors and for a Maxwellian gas as found in gaseous plasmas. It is interesting to note that for $z \gg 1$, $\phi(z)$ is finite for a Maxwellian distribution and infinitely small for a Fermi distribution. Helicons in a gaseous plasma would therefore seem to have far greater promise for studying the Cherenkov mechanism than helicons in solid-state plasmas.
The damping factor for the wave, $\frac{k_i}{k_T}$, can simply be found from equation 4.7, recalling that $N_i \ll N_r$ and using the binomial approximation, yielding:

$$k_i = \frac{\sqrt{\pi}}{2} \frac{\sin^2 \theta}{|\cos \theta|} \frac{\omega}{N_e} \Phi(z)$$  \hspace{1cm} (4.9)

Some insight into the physical phenomena involved in the damping can be gained by rearranging equation (4.9) and using $N_r = \frac{c}{v_\phi}$:

then $z = \frac{v_\phi}{v_T \cos \theta}$

and $k_i = \frac{\sqrt{\pi}}{2} \frac{\sin^2 \theta}{\cos^4 \theta} \frac{\omega}{v_T} \left(\frac{v_\phi}{v_T}\right)^3 \exp\left\{-\left(\frac{v_\phi}{v_T \cos \theta}\right)^2\right\}$  \hspace{1cm} (4.10)

As previously stated, $v_\phi / v_T \gg 1$ and since the exponential term is dominant, the damping is small although finite under the assumption of a plasma with electrons with a low thermal velocity. The Cherenkov mechanism occurs when $\omega = k_z v_z$ but the assumption of low electron thermal velocity implies $\omega \gg k v_T$.

Now since $k_z < k(k \neq k_z$ since $\theta 
eq 0$), for Cherenkov damping to occur

$v_z \gg v_T$.

This discussion has assumed a Maxwellian distribution of electron velocities and therefore it is only a small number of electrons in the high-energy tail of the distribution which contribute to the damping. In most laboratory plasmas the assumption of Maxwellian distribution is quite good, but in the case of plasmas formed by the interaction of R.F. fields it is the
electrons which are preferentially excited. There exists a small group of high-energy electrons which ionize the gas, coexisting with the lower temperature high density electrons constituting the bulk of the plasma.

The R.F. maintained plasma of L & T and the plasma described in this thesis approximate well to this model and for low values of the resistivity \((\text{high } \Omega_e \tau)\) the Cherenkov mechanism would be expected to be the dominant damping mechanism for the right-hand polarized waves.

In the American literature, this collisionless phenomena is known as magnetic Landau damping, or sometimes as transit time damping. As can be seen from equation 4.9, the damping is zero for propagation along the magnetic field \(B_0\). However, if the helicon is propagating at some non-zero angle \(\theta (\neq \pi/2)\) to \(B_0\), there are components of the wave fields along \(B_0\) and hence in the direction of the particle motion. As the energy of the helicon wave is dominantly stored in the wave magnetic fields, the damping due to particles interacting with the longitudinal electrical field is negligibly small (Buchsbaum and Platzman (1967)). The component of the helicon's magnetic field along \(B_0\) alternately adds to and subtracts from the applied magnetic field. Hence the local magnetic field in the plasma oscillates in magnitude as the electron moves along a line parallel to \(B_0\).

These ridges in the magnetic field act as "magnetic mirrors" which tend to restrict the particle motion along the field. When the electron velocity distribution is Maxwellian, there are more particles moving slower than the phase velocity of the wave than there are electrons moving faster (figure 4.7). Hence there is a nett transfer of energy to the particle system, resulting in attenuation of the wave (Houck and Bowers (1968)).
Fig. 4.7 Maxwellian distribution of electron velocities in the $z$-direction. The number of electrons absorbing energy from the wave are shown by the +ve signs while those giving energy to the wave are shown by the -ve signs.
As the angle θ is increased, the wave acquires more of a longitudinal character resulting in increasing damping as shown in equation 4.9.

The inclusion of thermal effects in a finite sized cylindrical plasma presents formidable theoretical problems in the calculation of the dispersion and attenuation of the waves. Formally, the refractive index can be considered as a vector, as the plasma in a magnetic field is anisotropic. The integral representation of Bessel functions can then be used to convert from plane to cylindrical geometry. The resultant refractive index will be a Bessel function with a complex argument. Rather than do the involved algebra associated with this problem, a qualitative argument will be presented in which waves in a cylindrical plasma can be considered as a superposition of plane waves which are damped according to equation 4.9.

The following method is due to Davies (1970), but is in quite common usage (e.g. Legéndy (1964)). We consider a plane wave propagating at some angle to the magnetic field $B_0$, with a wave vector $q$, where $q = (0, \gamma, k)$ so that $\gamma$ and $k$ can be regarded as transverse and longitudinal wave numbers respectively. Fields in a uniform cylindrical plasma are assumed to have the dependence $f(r) \exp[i(\omega t - k z - m \phi)]$. These can be obtained from the plane waves by rotating the vector $q$ about the $z$-axis (parallel to $B_0$) through an angle $\phi$, multiplying the fields by the factor $\exp(-im\phi)/2\pi$ and integrating over $\phi$ from $-\pi$ to $+\pi$. The integration over $\phi$ is performed by using the integral representation of Bessel functions (Lebedev (1965))

$$J_n(\rho) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp \left[ i(\rho \sin \xi - n \xi) \right] d\xi.$$
Consequently, propagation in a plasma cylinder can be considered as a superposition of plane waves travelling at an angle to the axial direction of the cylinder (and consequently to $B_0$). These waves will have fields with longitudinal components and hence can be damped by the electric or magnetic Landau mechanism. However, as mentioned previously, the helicon wave field is predominantly magnetic and the Cherenkov (magnetic Landau damping will be predominant).

This method allows the damping to be calculated provided $\omega$ is not too far from $\Omega_e$. In the case of helicon waves and compressional Alfvén waves, a full analysis is required to obtain an expression for damping. This has been done by Dolgopolov et al. (1963) for the right-hand wave in the helicon region, $\Omega_i \ll \omega \ll \Omega_e$, when the electron velocity $v_z$ along $B_0$ is near $\omega/k$, the phase velocity of the wave. Instead of using rigid physical boundaries in their plasma model, they consider a line parallel to $B_0$ with the plasma density decreasing radially from this line (i.e. $n(r)$). A rapidly changing $n(r)$ would define a plasma cylinder whereas $\frac{\partial n(r)}{\partial r} = 0$ would define an infinite plasma. An electric field $E_\parallel$ in the radial direction is produced by this charge distribution and is incorporated in the theory with the other constant field, the axial magnetic field $B_0$. By using Boltzmann's equation with the appropriate Ohms law for helicon waves, Dolgopolov et al. solve for the wave fields using a method of sequential approximation applied to the wave fields. Only first order terms are retained in the analysis. The helicon is assumed to be excited by an external current sheet and the expressions for the wave fields are used to obtain the flow of energy from the wave to the plasma electrons per unit axial length.
The absorption of the wave energy is found to increase strongly when \( \left( \frac{1}{\omega_p^2} \frac{3\omega_p^2}{\gamma(kr)} \right)^2 \) increases. Since the plasma frequency is a function of the charged particle density, this expression can be rearranged to yield

\[
\text{absorption} = \left[ \frac{A}{n(r)^2} \frac{\gamma(n(r))^2}{\gamma r} \right]^2
\]

where \( A \) is a constant depending on \( k, e, m_e, a, \) (the plasma radius) and \( n(r) \) is the radially varying plasma density.

It can be seen that for large \( \gamma(n(r))^2/\gamma r \), i.e. a cylindrical plasma with quite sharply defined boundaries, the absorption is large. The absorption disappears when \( \gamma(n(r))^2/\gamma r = 0 \), i.e. in an infinite plasma. This is to be expected as the wave would then be a plane wave propagating along \( B_0 \), and would be purely transverse in nature with no field components in the direction of propagation.

In discussing the damping of free oscillations in a plasma cylinder, rather than the forced oscillations considered in the section above, Dolgopolov et al. use a complex wave number \( k' = k + ik_1 \) and solve the equations neglecting the exciting currents outside the plasma. A simplified expression for the damping is derived for the assumptions of \( ak = 1 \), and \( v_\phi \ll v_T \), (c.f. \( v_\phi \gg v_T \) in section 4.2), conditions which prevail in the plasma discussed in this thesis. To obtain an order of magnitude approximation for the damping, the dispersion relation for helicons in an infinite collisionless medium is assumed:

\[
i.e. \quad k^2 = \frac{\omega_p^2}{\Omega_e} \omega,
\]
consequently,

\[ \frac{k_i}{k_r} \sim \frac{v_T k}{\Omega_e} \]

\[ \sim \frac{v_T \omega}{v_\phi \Omega_e} \]  \hspace{1cm} (4.11)

Comparing this expression with equation 4.9 it can be seen that the damping in the cylindrical case can remain significant for \( \omega \ll \Omega_e \), while for plane waves, the damping falls off exponentially as \( \omega \) becomes much smaller than \( \Omega_e \). The Cherenkov damping will therefore be expected to be the dominant collisionless mechanism for helicon waves in a cylindrical plasma. However, this damping becomes exponentially small for these frequency regions for plane wave propagation.

The results of this discussion will be used in chapter 7 in relation to the damping of a standing helicon wave.